RESONANCES: GEOMETRIC SCATTERING AND DYNAMICS
RÉSONANCES : SCATTERING GÉOMÉTRIE ET DYNAMIQUE

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ABSTRACTS

Alexander Adam

Resonances for Anosov diffeomorphisms

Abstract: Deterministic chaotic behavior of invertible maps T is appropriately described by the existence of expanding and contracting directions of the differential of T. A special class of such maps are Anosov diffeomorphisms. Every hyperbolic matrix M with integer entries induces such a diffeomorphism on the 2-torus. For all pairs of real-analytic functions on the 2-torus, one defines a correlation function for T which captures the asymptotic independence of such a pair under the evolution T^n as $n \to \infty$. What is the rate of convergence of the correlation as $n \to \infty$, e.g. what is its decay rate? The resonances for T are the poles of the Z-transform of the meromorphic continued correlation function. The decay rate is well-understood if $T = M$. There are no non-trivial resonances of M. In this talk I consider small real-analytic perturbations T of M where at least one non-trivial resonance of T appears. This affects the decay rate of the correlation.

Viviane Baladi

Linear response for discontinuous observables

Abstract: Linear response for hyperbolic dynamics is usually stated for smooth enough observables. Discontinuous observables (involving thresholds) appear naturally in extreme value theory. We present our recent results with Kuna and Lucarini giving sufficient conditions, on observables involving thresholds, ensuring linear response. Our proof uses the fine properties of anisotropic Banach spaces, and this will also be an opportunity to give a survey talk on anisotropic spaces suitable for transfer operators of hyperbolic dynamical systems.

Ben Bellis

Resolvent estimates for non-self-adjoint semiclassical Schrd̈inger operators

Abstract: Understanding the resolvent of a differential operator is often an important step in understanding various functions of that operator. For self-adjoint operators, the spectral theorem provides a very powerful tool to estimate the norm of the resolvent, but there is no suitable analog in the non-self-adjoint case. This makes resolvent estimates for non-self-adjoint operators generally more difficult to attain. In this talk we provide a bit of background on the estimating resolvent for such operators in the semiclassical setting and outline a proof of one such estimate.

Yannick Bonthonneau

Ruelle resonances for cusps

Joint work with T. Weich.

Abstract: I will explain how we can use anisotropic spaces – in Dyatlov-Zworski's formulation – and Melrose's b-calculus to prove the existence of Ruelle spectrum for manifolds with hyperbolic cusps. If time allows, I will also talk about zeta functions and scattering matrices.

David Borthwick

Asymptotics of Resonances for Hyperbolic Surfaces

Abstract: After a brief introduction to the spectral theory of hyperbolic surfaces, we will focus on the problem of understanding the asymptotic distribution of resonances for hyperbolic surfaces. The theory of open quantum chaotic systems has inspired several interesting conjectures about this distribution. We will highlight the recent theoretical progress towards these conjectures, and present some of the latest numerical evidence.

Nguyen Viet Dang

Resonances of Morse gradient flows and the Witten complex

Abstract: Consider a flow generated by a gradient vector field V on some compact Riemannian manifold satisfying some generic transversality condition. We explain how to derive the correlation spectrum of the flow by studying the spectral properties of the Lie derivative $-L_V$ acting on appropriate anisotropic Sobolev spaces of currents. Then we will show how our spectral methods allow to realize the Thom-Smale-Witten complex in the space of currents.

Alexis Drouot

Pollicott–Ruelle resonances via kinetic Brownian motion

Abstract: We give a probabilistic definition of Pollicott–Ruelle resonances for chaotic (Anosov) geodesic flows. These complex numbers are usually defined as dynamical quantities that quantize the decay of classical correlations. In this work, we realize them as the small white-force limits of the eigenvalues of the generator of the kinetic Brownian motion (a random perturbation of the geodesic flow). The proof requires semiclassical hypoelliptic estimates and specifically designed anisotropic Sobolev spaces.

Frédéric Faure

Fractal upper bound for the density of Ruelle spectrum of Anosov flows

Abstract: We consider a smooth vector field X on a closed manifold M that generates an Anosov flow. Let $V \in C^{\infty}(M;\mathbb{R})$ be a smooth potential function. It is known that for any $C > 0$, there exists some anisotropic Sobolev space $\mathcal{H}_C$ such that the operator $A = -X + V$ has intrinsic discrete spectrum on $\mathrm{Re}(z) > -C$. The discrete eigenvalues are called Ruelle-Pollicott resonances [Butterley-Liverani 2007, Faure-Sjöstrand 2011] and the density of eigenvalues is bounded by $O(\langle\omega\rangle^{n})$ with $\omega = \mathrm{Im}(z)$ and $n = \dim M - 1$. We will present a better upper bound, that is $O\left(\langle\omega\rangle^{\frac{n}{1+\beta_0}}\right)$ where $0 < \beta_0 \leqslant 1$ is the Hölder exponent of the distribution $E_u \oplus E_s$ (strong stable and unstable). For this we construct some specific anisotropic Sobolev spaces and use some micro-local analysis based on wave packet transform. We also obtain some new results concerning the wave front set of the resonances. Collaboration with Masato Tsujii.

Charles Hadfield

Quantum resonances on asymptotically hyperbolic manifolds

Abstract: I will show how a technique presented in Andras Vasy's talk provides the meromorphic extension of the resolvent of the Laplacian on vector bundles. This is a key lemma in generalizing the quantum classical correspondence of Tobias Weich's talk to convex co-compact hyperbolic manifolds where symmetric tensors and the Lichnerowicz Laplacian play an important role.

Luc Hillairet

Spectral determinant for Hurwitz and Mandelstam diagrams

Abstract: Hurwitz and Mandelstam diagrams describe meromorphic functions or one-forms on a compact Riemann surface. It is possible to associate with each setting a flat metric with conical singularities and cylindrical, conical or flat Euclidean ends. Using scattering methods, we will explain how to construct a regularized determinant for the flat Laplacian. We will then address how this determinant varies when the diagram changes.

Joint work with A. Kokotov and V. Kalvin.

Peter Hintz

The stability of Kerr-de Sitter black holes

Abstract: In this lecture I will discuss Kerr-de Sitter black holes, which are rotating black holes in a universe with a positive cosmological constant, i.e. they are explicit solutions (in 3+1 dimensions) of Einstein's equations of general relativity. They are parameterized by their mass and angular momentum.

I will discuss the geometry of these black holes, and then talk about the stability question for these black holes in the initial value formulation. Namely, appropriately interpreted, Einstein's equations can be thought of as quasilinear wave equations, and then the question is if perturbations of the initial data produce solutions which are close to, and indeed asymptotic to, a Kerr-de Sitter black hole, typically with a different mass and angular momentum. In this talk, I will emphasize geometric aspects of the stability problem, in particular showing that Kerr-de Sitter black holes with small angular momentum are stable in this sense.

Maxime Ingremeau

The scattering matrix and its spectrum in the semiclassical limit

Abstract: Consider a Schrödinger operator $P_h = -h^2\Delta + V$, where $V \in C_c^\infty(\mathbb{R}^d)$. A solution of $P_h f = f$ may always be written as the sum of an incoming and an outgoing part. The scattering matrix is the operator which maps the incoming part to the outgoing part. We will describe some of the properties of S_h, and of its spectrum, in the semiclassical limit $h \to 0$.

Long Jin

Resonances for Open Quantum Map

Abstract: Open quantum maps are useful models in the study of scattering resonances, especially for open quantum chaotic systems. In this talk, we discuss a special family of open quantum maps known as quantum open baker?s maps. They are quantizations of the open baker?s map on the torus and given by a family of subunitary matrices. We are interested in the distribution of eigenvalues, which are analogues of scattering resonances in this simple setting. In particular, we show that there exists a spectral gap which improves both the trivial gap and the pressure gap. We also show a fractal Weyl upper bound for the number of eigenvalues in annuli. This is joint work with Semyon Dyatlov.

Benjamin Küster

Ruelle resonances on homogeneous vector bundles

Abstract: Motivated by the result of Dyatlov, Faure, and Guillarmou that higher band Ruelle resonances on compact quotients of hyperbolic space can be identified with generalized first band Ruelle resonances on certain vector bundles, we define and study Ruelle resonances on general homogeneous vector bundles over compact rank one locally symmetric spaces. The main

goal is to identify these resonances with quantum eigenvalues of a generalized Laplacian via Olbrich's vector valued Poisson transform, generalizing the known results for higher band Ruelle resonances from compact quotients of hyperbolic space to general compact locally symmetric spaces of rank one. - Joint work with Tobias Weich.

Roberto Miatello

Spectra on p-forms of lens spaces from norm one length-spectra of congruence lattices

Abstract: We will relate the multiplicity of eigenvalues of the Hodge-Laplace operator on p-forms with the so called norm-one*-length spectra of an associated congruence lattice. As a consequence we will give families of Hodge-isospectral pairs that are not strongly isospectral. We will also describe a connection with the geometry of toric manifolds.

Werner Müller

Dynamical zeta functions of locally symmetric spaces of finite volume

Abstract: The dynamical zeta functions that I will consider in this talk are functions of a complex variable which are defined in terms of the length spectrum of closed geodesics. They provide a bridge between dynamical and spectral data. For a hyperbolic surface they were introduced by Selberg and Ruelle. The aim of the talk is to discuss recent results concerning dynamical zeta functions of locally symmetric spaces of finite volume. In the non-compact case scattering theory comes into play.

Frédéric Naud

Resonances in the large p limit

Abstract: Given a convex co-compact subgroup Γ of $SL_2(\mathbb{Z})$, one can define "congruence subgroups" $\Gamma(p)$ associated to a square free or a prime number p. These groups play a key role in various number theoretic problems such as finding almost primes in thin subgroups. In this talk we investigate the behavior of resonances of the associated hyperbolic surfaces "in the large p regime". In particular we will discuss questions related to Weyl laws and behaviour of resonances close to the unitary axis.

Aprameyan Parthasarathy

Boundary values, resonances, and scattering poles (rank-one case)

Abstract: In this talk, we report on joint work with (Sönke Hansen and Joachim Hilgert). We give a definition of boundary values for tempered eigenfunctions of the Laplacian on a Riemannian symmetric space of rank one, following Oshima's simplified definition of boundary values. We then use this to give a new proof of the distributional Helgason conjecture. We apply this to show that the the scattering kernel is given by taking boundary values of the resolvent kernel in both the variables. Further, we use this to show that for generic parameters the boundary value map gives an isomorphism between the ranges of the resolvent and the scattering residue operators. This is analogous to the result obtained by Borthwick and Perry for asymptotically hyperbolic manifolds.

Anke Pohl

Isomorphisms between eigenspaces of slow and fast transfer operators

Abstract: Over the last few years I developed (partly jointly with coauthors) dual 'slow/fast' transfer operator approaches to automorphic functions, resonances, and Selberg zeta functions for certain hyperbolic surfaces/orbifolds $L \setminus H$ with cusps (both of finite and infinite area; arithmetic and non-arithmetic).

Both types of transfer operators arise from discretizations of the geodesic flow on $\Gamma \backslash \mathbb{H}$. The eigenfunctions with eigenvalue 1 of slow transfer operators characterize Maass cusp forms. Conjecturally, this characterization extends to more general automorphic functions as well as to residues at resonances. The Fredholm determinant of the fast transfer operators equals the Selberg zeta function, which yields that the zeros of the Selberg zeta function (among which are the resonances) are determined by the eigenfunctions with eigenvalue 1 of the fast transfer operators. It is a natural question how the eigenspaces of these two types of transfer operators are related to each other.

In this talk I will discuss an isomorphism between these eigenspaces. This is joint work with Alexander Adam.

Mark Pollicott

Dynamical zeta functions and validated numerical computation

Abstract: There are many numerical quantifiers of a dynamical system which can be read off from the properties of a suitable zeta function. We will consider some instances of this, with an emphasis on obtaining rigorous estimates on their numerical values.

Gabriel Rivière

Correlation spectrum of Morse-Smale flows

Abstract: I will explain how one can get a complete description of the correlation spectrum of a Morse-Smale flow in terms of the Lyapunov exponents and of the periods of the flow. I will also discuss the relation of these results with differential topology.

This a joint work with Nguyen Viet Dang (Université Lyon 1).

Alexander Strohmaier

Generalised Analytic Functions and Applications to Scattering Theory

Abstract: I will introduce a generalisation of the field of meromorphic functions that includes the logarithm and real powers near the origin. I will discuss some properties of these functions and applications to scattering theory on symmetric spaces of even rank.
(based on joint work with J. Müller)

Andras Vasy

Microlocal analysis for Kerr-de Sitter black holes

Abstract: In this lecture I will describe a framework for the Fredholm analysis of non-elliptic problems both on manifolds without boundary and manifolds with boundary, with a view towards wave propagation on Kerr-de-Sitter spaces, which is the key analytic ingredient for showing the stability of black holes (see Peter Hintz' lecture). This lecture focuses on the general setup such as microlocal ellipticity, real principal type propagation, radial points and generalizations, as well as (potentially) normally hyperbolic trapping, as well as the role of resonances.

Tobias Weich

Classical and quantum resonances on hyperbolic surfaces

Abstract: For compact and for convex co-compact oriented hyperbolic surfaces, we give a complete description of the classical Ruelle resonances for the geodesic flow and their associated resonant states: The classical resonances are mostly given by shifted copies of quantum resonances. Additionally, at the negative integers, there occur "topological" Ruelle resonances and their multiplicity is purely determined by the Euler characteristic of the surface.

Maher Zerzeri

Asymptotic of resonances created by a multi-barrier potential

Abstract: In this talk, I will give the quantization condition and the semiclassical distribution of resonances of the Euclidean Schrödinger operator in a general setting where the trapped set of the underlying classical mechanics makes a finite graph consisting of hyperbolic fixed points and associated homoclinic/heteroclinic trajectories. I will give some examples and a rough sketch of the proof. Based on joint work with J-F. Bony, S. Fujiié and T. Ramond.

Asymptotique des résonances créées par un potentiel à multi-barrières

Résumé : Dans cet exposé, nous donnerons la condition de quantification et la distribution des résonances semiclassiques d'opérateurs de Schrödinger sur $L^2(\mathbb{R}^N), N \geqslant 1$; dans le cas où l'ensemble des trajectoires captées du hamiltonien associé est un graphe fini constitué de points fixes hyperboliques et des trajectoires homoclines/hétéroclines correspondantes. Nous donnerons deux ou trois exemples et quelques idées de preuve. Il s'agit d'un travail en collaboration avec J-F. Bony, S. Fujiié et T. Ramond.

Jupiter (Beite) Zhu

Normal forms of Pseudodifferential operators on Lagrangian submanifolds of radial points

Abstract: This talk aims to present a result from Nicholas Haber's thesis. Given a smooth n dimensional manifold X and a conic Lagrangian submanifold $\Lambda \to T^*X\backslash o$, we study the family of order m pseudodifferential operators with homogeneous real principal symbol p and with Hamilton vector field H_p that is radial and nonvanishing on Λ. Under the coordinate $\mathbb{R}_z \times \mathbb{R}_y^{n-1}$ and their dual variables ζ, η, the main result states that such operator at $q \in \Lambda$ is microlocally equivalent to $zD_z + p_0(y)$ at $q_0 = (y = 0, z = 0, \eta = 0, \zeta = 1)$ for some $p_0 \in C^\infty(\mathbb{R}^{n-1})$. The proof first tries to show the equivalence on the principal symbol level, adopting Nelson's proof of the Sternberg linearization theorem, then proceeds to eliminate errors at lower order terms by a formal power series argument.