Resonances of the Laplacian for Riemannian symmetric spaces

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In these lectures we will study the resonances of the Laplacian Δ of a Riemannian symmetric space of the noncompact type $X = G/K$. The easiest examples of these spaces are the real hyperbolic spaces, where $G = SO_0(n,1)$ is the connected component of the special orthogonal group of signature $(n,1)$ and $K = SO(n)$.

Let $\sigma(\Delta)$ denote the spectrum of Δ. Then the resolvent $R(z) = (\Delta - z)^{-1}$ is a holomorphic function on $\mathbb{C} \setminus \sigma(\Delta)$, with values in the space of bounded linear operators on $L^2(X)$. If R admits a meromorphic continuation of as a distribution valued map on a Riemann surface above $\mathbb{C} \setminus \sigma(\Delta)$, then the poles of the meromorphically extended resolvent are the resonances of Δ.

Nowadays there is no global result on the existence and nature of resonances on general Riemannian symmetric spaces of the noncompact type. In some examples the extensions turns out to be holomorphic and there are no resonances, as in the case when G admits a complex structure. In other examples, the resonances exist and can be explicitly determined. They are then linked to the spherical principal series representations of G.

We present the approach to resonances on Riemannian symmetric spaces which is based on Fourier analysis, more precisely on the so-called Fourier-Helgason transform. After having recalled the necessary structural properties of G/K, we will outline the most important tools for the L^2 harmonic analysis on G/K, that is the Paley-Wiener and the Plancherel theorems. This will allows us to write an explicit integral formula for the resolvent. Understanding the resonances of the Laplacian means understanding the meromorphic continuation of this singular integral.

References

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