Section 2.3: Modeling with First Order Equations

Objectives:

1. Model a real-world situation with a DE.
2. Solve the DE.
3. Analyze and interpret the solutions.

Example 1: Water Tank

A tank initially contains 40 pounds of salt dissolved in 600 gallons of water. Starting at time $t = 0$, water that contains 1/2 pound of salt per gallon is poured into the tank at the rate of 4 gal/min and the mixture is drained from the tank at the same rate.

(a) Construct a differential equation for $Q(t)$, which is the number of pounds of salt in the tank at time $t > 0$.

(b) Solve the DE to determine an expression for $Q(t)$.

(c) After a long period of time, what happens to the concentration of salt in the tank?

Example 2: Population Model

The world population in 2019 is roughly 7.7 billion.

(a) The world population is increasing at a rate of 1.1% per year. If the growth rate remains fixed at 1.1%, how long will it take for the population of the world to reach 20 billion people?

(b) Assume the earth cannot support a population beyond 20 billion people. If the population growth rate is **also** proportional to the difference between how close the world population is to this limiting value, what is the expression that gives the world population as a function of time?