How to guess the form of $Y(t)$:

- If $g(t) = e^{\alpha t}$, then choose $Y(t) = At^s e^{\alpha t}$,
- If $g(t) = \sin(\beta t)$ or $\cos(\beta t)$, then choose $Y(t) = t^s(A\sin(\beta t) + B\cos(\beta t))$,
- If $g(t) = R(t)$ is a polynomial of degree n, then choose $Y(t) = t^s S(t)$ a polynomial of degree n.
- If $g(t)$ is a product of any two or all three of the above functions, then choose Y to be the corresponding product as above.
- The parameter s can be $0, 1, 2$. It is the *smallest* s among $0, 1, 2$ so that the guess for $Y(t)$ is not a solution of the corresponding homogeneous DE.
- If $g(t) = g_1(t) + \cdots + g_r(t)$, then determine a particular solution $Y_j(t)$ for each differential equation $ay'' + by' + c = g_j(t)$ for $j = 1, \ldots, r$. Set $Y = Y_1 + \cdots + Y_r$.

Superposition principle for nonhomogenous DE

The last step for guessing the form of the particular solution was:

If $g(t) = g_1(t) + \cdots + g_r(t)$, then determine a particular solution $Y_j(t)$ for each differential equation $ay'' + by' + c = g_j(t)$ for $j = 1, \ldots, r$. Set $Y = Y_1 + \cdots + Y_r$.

It is a special instance (for constant coefficient linear DE) of the following general principle.

Theorem (Principle of superposition)

Consider the linear differential equation $L[y] = g(t)$.

Suppose that the function $g(t)$ splits into two functions $g(t) = g_1(t) + g_2(t)$.

If Y_1 is a solution of $L[y] = g_1(t)$ and Y_2 a solution of $L[y] = g_2(t)$ then $Y_1 + Y_2$ is a solution of $L[y] = g(t)$.

Example:
Write the form the general solution of

$$y'' - 3y' - 4y = e^{-t} - 4t^2$$