- $y'' - 3y' - 4y = e^{-t} - 4t^2 \qquad (*)$

The general solution is of the form $y(t) = y_c(t) + Y_1(t) + Y_2(t)$ where

- $y_c(t) = C_1 e^{4t} + C_2 e^{-t}$ is the complementary solution
- $Y_1(t) = -\frac{1}{5} t e^{-t}$ is the particular solution of $y'' - 3y' - 4y = e^{-t}$ found before
- $Y_2(t)$ is a particular solution of $y'' - 3y' - 4y = -4t^2$, which we are going to determine. Since t^2 is not in $y_c(t)$, the table suggests $Y_2(t)$ will be of the form $Y_2(t) = At^2 + Bt + C$ (= polynomial of same degree as t^2). We are going to determine A, B, C by substituting $Y_2(t)$ in the DE

$Y_2' = 2At + B$

$Y_2'' = 2A$

$$Y_2'' - 3Y_2' - 4Y_2 = -4t^2 \iff 2A - 3(2At + B) - 4(At^2 + Bt + C) = -4t^2$$

$$\iff -4At^2 - (6A + 4B)t + 2A - 3B - 4C = -4t^2$$

$$\iff \begin{cases} -4A = -4 \\ 3A + 2B = 0 \\ 2A - 3B - 4C = 0 \end{cases} \iff \begin{cases} A = 1 \\ B = -3/2 \\ C = \frac{1}{4}(2A - 3B) = \frac{1}{4}\left(2 + \frac{9}{2}\right) = \frac{13}{8} \end{cases}$$

Hence: $Y_2(t) = t^2 - \frac{3}{2}t + \frac{13}{8}$

Conclusion: the general solution of the DE $(*)$ is

$y(t) = C_1 e^{4t} + C_2 e^{-t} - \frac{1}{5} t e^{-t} + t^2 - \frac{3}{2}t + \frac{13}{8}$, where C_1, C_2 are arbitrary constants.