# Section 5.6: Differential equations with discontinuous forcing functions

## Main Topics:

Examples of differential equations with constant coefficients

$$ay'' + by' + cy = f(t)$$

in which the nonhomogenous term $f$ (=the forcing function in a spring-mass system) is **not continuous**.

**General fact:** even if $f$ is not continuous but **piecewise continuous**, then the solutions of the DE are continuous.

This fact also hold for constant coefficient DEs of order > 2.

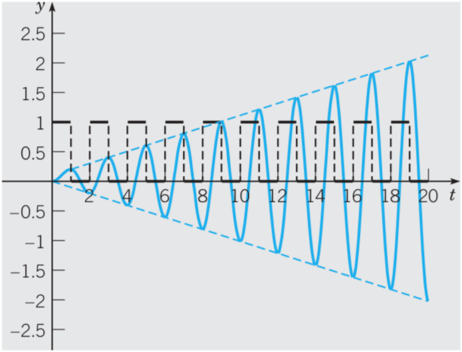
**Example 1:**

Using the Laplace transform, solve the initial value problem

$$y'' + \pi^2 y = f(t) \quad \text{with} \quad y(0) = 0,\, y'(0) = 0 \quad \text{where} \quad f(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ 0 & \text{if } 1 \le t < 2 \end{cases}$$

and $f(t)$ has period 2.

Then draw the graphs of the solution and the forcing function, explain how they are related.




$y(t)$ is increasing when $f(t) = 1$,
$y(t)$ is decreasing when $f(t) = 0$.

The phase synchronization between $f(t)$ and $y(t)$ originates a phenomen of resonance.

**Example 2:**

Using the Laplace transform, solve the initial value problem

$$y'' + 3y' + 2y = u_2(t) \quad \text{with} \quad y(0) = 6,\ y'(0) = 6.$$

Then draw the graphs of the solution and the forcing function, explain how they are related.