SECTION 6.1

EXAMPLE Verify that $\mathbf{X}(t)=e^{-t}\begin{pmatrix}6\\-8\\-4\end{pmatrix}+2e^{2t}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ satisfies the system of linear DEs:

$$\mathbf{X}'=\begin{pmatrix}1&1&1\\2&1&-1\\0&-1&1\end{pmatrix}\mathbf{X}$$

LHS: $\mathbf{X}'(t)=-e^{-t}\begin{pmatrix}6\\-8\\4\end{pmatrix}+4e^{2t}\begin{pmatrix}0\\1\\-1\end{pmatrix}$

RHS: $\begin{pmatrix}1&1&1\\2&1&-1\\0&-1&1\end{pmatrix}\left[e^{-t}\begin{pmatrix}6\\-8\\-4\end{pmatrix}+2e^{2t}\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]=$

$$=-e^{-t}\begin{pmatrix}1&1&1\\2&1&-1\\0&-1&1\end{pmatrix}\begin{pmatrix}6\\-8\\-4\end{pmatrix}+2e^{2t}\begin{pmatrix}1&1&1\\2&1&-1\\0&-1&1\end{pmatrix}\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

$$=-e^{-t}\begin{pmatrix}-6\\8\\4\end{pmatrix}+\underbrace{2e^{2t}\begin{pmatrix}0\\2\\-2\end{pmatrix}}_{4e^{2t}\begin{pmatrix}0\\1\\-1\end{pmatrix}}$$

Hence: LHS = RHS

EXAMPLE Transform $y^{(4)}+6y'''+3y=t$ into an equivalent system of linear DE:

$x_1=y,\ x_2=y',\ x_3=y'',\ x_4=y^{(3)} \Rightarrow \begin{cases}x_1'=y'=x_2\\ x_2'=y''=x_3\\ x_3'=y'''=x_4\\ x_4'+6x_4+3x_1=t\end{cases}$

i.e. $\begin{cases}x_1'= & x_2\\ x_2'= & x_3\\ x_3'= & x_4\\ x_4'=-3x_1 & -6x_4+t\end{cases}$ i.e. $\mathbf{X}'(t)=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-3&0&0&-6\end{pmatrix}\mathbf{X}(t)+\begin{pmatrix}0\\0\\0\\t\end{pmatrix}$