EXAMPLE (SECTION 4.2)

Determine the longest interval in which the IVP

$$(t^2-1)y'' - 3ty' + 4y = \sin(t) \quad \text{with} \quad y(0)=2,\ y'(0)=1$$

has a twice diff. solution.

Solution The standard form of the DE is $y'' - \underbrace{\frac{3t}{t^2-1}}_{p(t)} y' + \underbrace{\frac{4}{t^2-1}}_{q(t)} y = \underbrace{\frac{\sin t}{t^2-1}}_{g(t)}$, where p, q and g are continuous functions of t for all $t \neq \pm 1$. The largest interval containing 0 on which they are continuous is therefore $(-1;1)$. This is the largest interval where the given IVP has a twice differentiable solution.

EXAMPLES (SECTION 4.2)

- (Ex. 4.2, nº 12) Find the Wronskian of the functions x and xe^x

$$W[x, xe^x] = \begin{vmatrix} x & xe^x \\ 1 & e^x(1+x) \end{vmatrix} = xe^x(1+x) - xe^x = x^2e^x$$

- (Ex. 4.2, nº 18 If $W[f,g](t) = 3e^{2t}$ and $f(t) = e^{4t}$, find $g(t)$

$$3e^{2t} = \begin{vmatrix} e^{4t} & g(t) \\ 4e^{4t} & g'(t) \end{vmatrix} = e^{4t}(g'(t) - 4g(t))$$

i.e. $g'(t) - 4g(t) = 3e^{-2t}$ (1st order linear DE in g)

Integrating factor $\mu(t) = e^{-4t}$ because $\int -4\,dt = -4t + C$

$$(e^{-4t}g(t))' = 3e^{-2t}e^{-4t} = 3e^{-6t}$$

$$e^{-4t}g(t) = \int 3e^{-6t}\,dt = \frac{1}{2}e^{-6t} + C$$

$$(te^{-t})' = e^{-t} - te^{-t} = e^{-t}(1-t) = -t$$

So: $g(t) = \frac{1}{2}e^{-2t} + Ce^{4t}$, $C \in \mathbb{R}$