Georgia Tech Lorraine - Metz — SPRING 2019
MATH 2552 — Differential Equations
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MIDTERM I

____ (Name and Signature)

1. Consider the following initial value problem

$$y' + y^3 \sin(x) = 0 \text{ and } y(0) = 3$$

a. (2 points) Identify the equation (order, linear, homogeneous, nonlinear, separable, exact, etc). *(Explain and justify your reasoning.)*

b. (5 points) Solve the initial value problem. *(You may leave your answer in an implicit form.)*

2. Consider the matrix $\mathbf{A} = \begin{pmatrix} -5 & 2 \\ -6 & 2 \end{pmatrix}$

a. (3 points) Compute the trace, the determinant and the characteristic equation of $\mathbf{A}$.

b. (4 points) Solve the system of differential equations: $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

c. (1 point) Explain why $(x, y) = (0, 0)$ is the unique critical point of the system.

3. (3 points) Determine the longest interval where the solution of the following initial value problem exists and is unique. Justify your answer. *(Do not attempt to solve the differential equation.)*

$$(t^2-4)\frac{dy}{dt}+3\ln(t)y=5\sin(t^2) \text{ with } y(1)=5$$

5. (4+3 points) Two tanks are connected: a pipe where the water from Tank 1 goes into Tank 2. Initially Tank 1 contains 20 gal of water and 10 oz of sugar, while Tank 2 contains 40 gal of water and 20 oz of sugar. Water containing 5 oz/gal of sugar flows into Tank 1 at a rate of 3 gal/min, and the well-stirred solution flows from Tank 1 to Tank 2 at a rate of 3 gal/min. The well stirred solution in Tank 2 drains out at a rate of 3 gal/min and leaves the system. Denote by $Q_1(t)$ the amount (in oz) of sugar in Tank 1 at time t, and $Q_2(t)$ the amount (in oz) of sugar in Tank 2 at time t.

Set up the system of differential equations that models the amount of sugar in each tank: clearly write the system of the form $\mathbf{Q}'=\mathbf{A}\mathbf{Q}+\mathbf{b}$ where $\mathbf{Q}=\begin{pmatrix}Q_1\\Q_2\end{pmatrix}$. *(No need to find the solution.)*

Find the equilibrium of this system.