

Last Name:
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## Midterm n<sup>0</sup> 2 (50 minutes)

*Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 25 points*

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**Exercise 1** (a) [3 points] Find the general solution of the homogenous linear differential equation

$$y'' - 6y' + 8y = 0.$$

(b) [5 points] Find a particular solution of the linear differential equation

$$y'' - 6y' + 8y = 2t + e^{2t}.$$

Exercise 1 (continued)

(c) [4 points] Solve the initial value problem

$$y'' - 6y' + 8y = 2t + e^{2t}, \quad y(0) = 0, y'(0) = 0.$$

**Exercise 2** Let  $f$  be the function defined on  $[0, +\infty)$  by

$$f(t) = \begin{cases} e^{t/2}, & 0 \leq t \leq 2 \\ 1 & t > 2 \end{cases}$$

(a) [2 points] Sketch the graph of  $f$ .

(b) [2 points] Show that  $f$  is piecewise continuous.

(c) [2 points] Show that  $f$  is of exponential order (as  $t \rightarrow +\infty$ )

Exercise 2 (continued)

(d) [4 points] Using the definition, compute the Laplace transform of  $f$ .

(e) [2 points] Compute  $\lim_{s \rightarrow +\infty} \mathcal{L}\{f\}(s)$ .

Why should we know this limit without computing it? Justify your answer.

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**Exercise 3** [2 points]

Is the function  $f(t) = e^{t(t-1)}$  of exponential order? Justify your answer.