# Review sheet for Midterm 1

## Chapter 1: Introduction

*Section 1.1:* use differential equations for modeling. Initial value problems (IVP).
*Section 1.2 and Section 2.5:* Autonomous differential equations: find equilibrium solutions (also called critical points or stationary points), draw phase lines, sketch integral curves. Determine if a critical point is asymptotically stable, semistable or unstable. (drawing of direction fields is not requested).
*Section 1.3:* classification of differential equations: order, linear/non linear, homogeneity

## Chapter 2: First order differential equations

*Section 2.1:* solve separable equations.
*Section 2.2:* standard form of a first order linear DE. Solve first order linear differential equations by using integrating factors.
*Section 2.3:* modeling: write down a differential equation to model a problem and then solve the differential equation (or the IVP).
*Section 2.4:* existence and uniqueness of solutions: first order linear DE (Theorem 2.4.1) and first order non-linear DE (Theorem 2.4.2).
*Section 2.5:* see Section 1.2.
*Section 2.6:* recognize a first order exact DE and solve it.

## Chapter 3: Systems of two first order equations

*Section 3.1:* System of two linear equations in matrix form. Trace and determinant of a $2 \times 2$ matrix. A matrix is invertible if and only if its determinant is non-zero. Inverse of a matrix. Solutions of linear systems. Eigenvalues and eigenvectors.
*Section 3.2:* IVP for a system of two first-order linear DE's. Matrix form. Component plots of solutions. Homogenous systems.
Autonomous systems: notions of phase plane, trajectories, direction fields, equilibrium point (or equilibrium point or critical point), phase portrait.
Transform a second order linear DE into a system of first order linear DE's.
*Section 3.3:* Reduce the non-homogeneous system $\mathbf{x}' = A\mathbf{x} + b$ to the homogeous s system $\mathbf{x}' = A\mathbf{x}$.

For the homogeneous system $\mathbf{x}' = A\mathbf{x}$:
Superposition principle (Theorem 3.3.1), Wronskian and linear independence, fundamental system of solutions, general solution (Theorem 3.3.4).
*Sections 3.3 and 3.4:* Solve the homogeneous system $\mathbf{x}' = A\mathbf{x}$:

- Find a fundamental system of solutions and write the general solution (depending of the nature of the eigenvalues of $A$).
- When A has complex eigenvalues, write the solution in terms of real solutions (Section 3.4).
- Draw phase portraits when $A$ has two distinct real eigenvalues (Table 3.3.1) or complex eigenvalues (Table 3.4.1).
- Determine if (0, 0) is a nodal sink, a nodal source, a saddle a spiral sink, a spiral source, or a center.

*Sections 2.7, 3.4, 3.5, 6.1 and 6.2 are not on Midterm 1.*