Review sheet for Midterm 2

Chapter 4: Second order linear equations

Section 4.1: Second order linear equations: standard form, homogeneous/nonhomogenous, constant coefficients/variable coefficients, initial value problems (IVP). See below for the modelisation of a spring-mass system.

Section 4.2: The system of first order linear differential equations associated with a second order linear differential equation, correspondence of initial conditions, matrix notation.
Existence and uniqueness of the solutions of an IVP for a 2nd order linear DE (Theorem 4.2.1).
Second order linear homogenous differential equations: differentiation and multiplication by a function as linear operators, principle of superposition for a 2nd order DE (Theorem 4.2.2, Corollary 4.2.3) and for a homogenous system of 1st order linear DEs (Theorem 4.2.4, Corollary 4.2.5). Wronskian of two solutions, fundamental solutions and general solution: for homogenous systems of two 1st order linear DEs (Theorem 4.2.6) and for 2nd order linear DE (Theorem 4.2.7).

Section 4.3: Second order linear homogenous differential equations with constant coefficients: characteristic equation, fundamental system of solutions constructed from the roots of the characteristic equation (Theorem 4.3.1, for the DE and for its associated system), general solution (Theorem 4.3.2).
Remark: the subsection on IVPs and phase portraits (p. 233 to the end of section 4.2) is not in the program for Midterm 2.

Section 4.5: Second order linear nonhomogenous differential equation: form of the general solution as a sum of the general solution of the corresponding homogenous differential equation (complementary solution) and one particular solution (Theorems 4.5.1 and 4.5.2).
Methods for finding a particular solution:

- the method of undetermined coefficients for the constant coefficient case.
- the method of variation of parameters in the general case (see section 4.7 below).

Section 4.7: The method of variation of parameters for finding a particular solution of:

- a nonhomogenous system of 1st order linear DEs (Theorem 4.7.1)
- a 2nd order linear nonhomogenous DE (Theorem 4.7.2)

Remark: you are requested to understand how to compute the formulas for $\mathbf{x}_p$ in Theorem 4.7.1 and for Y in Theorem 4.7.2, but you are not requested to memorize these formulas.

Sections 4.1, 4.4 and 4.6: Spring-mass systems
Section 4.1: the model: mass, spring constant, damping factor.
Section 4.4: unforced or free systems (harmonic oscillators).
Undamped free system: phase-amplitude form of the general solution (period, natural frequency, phase, amplitude).
Damped free system: underdamped, critically damped or overdamped harmonic motion; critical damping; quasi-frequency and quasi-period of an underdamped harmonic motion.
Remark: the phase portraits for harmonic oscillators (pp. 249–250) are not in the program for Midterm 2.
Section 4.6: forced systems with periodic external force. The method of complex-valued exponentials $f(t) = Ae^{i\omega t}$. Transient solutions, steady-state solutions, frequency response function, gain factor, phase shift, resonance. The special case of forced systems without damping.

Chapter 5: The Laplace transform

Section 5.1: Improper integrals: exemples and tests of convergence (Theorem 5.1.4). Piecewise continuous functions. Functions of exponential order. The Laplace transform: definition, linearity (Theorem 5.1.2), Laplace transform of piecewise continuous functions, Laplace transforms of piecewise continuous functions of exponential order (Theorem 5.1.6, Corollary 5.1.7).

Section 5.2: Laplace transforms of $e^{ct}f$, of derivatives, of differential equations.