Georgia Tech – Lorraine
Spring 2020
Differential Equations
Math 2552
2/12/2020

Last Name:
First Name:

EX	
1	
2	
3	
4	
TOT	

Midterm n^0 1 (50 minutes)

- *Write your answers in the provided answer box.*
- *Show your work and justify your answers.*
- *Calculators, notes, cell phones, books are not allowed.*
- *Please do not use red or pink ink.*

Maximum: 25 points

Exercise 1 [1+1+1+1+5 points]
Consider the differential equation: $y' = x^2y^2 - 4x^2$.

(a) Is this differential equation separable? ANSWER: Yes ☐ No ☐.

Justify:

(b) Is this differential equation linear? ANSWER: Yes ☐ No ☐.

Justify. If it is linear, write it in standard form.

(c) Is this differential equation exact? ANSWER: Yes ☐ No ☐.

Justify:

(d) Is this differential equation a Bernoulli differential equation? ANSWER: Yes ☐ No ☐.

Justify:

Please turn: Question (e) on the following page ⟶

Exercise 1 (continued)

(e) Solve the differential equation $y' = x^2y^2 - 4x^2$.

[*Hint:* you might want to use a partial fraction decomposition to compute one of the integrals]

ANSWER:

$y =$

Exercise 2 **[3 points]** The size of a population of wolves is modeled by the differential equation

$$\frac{dy}{dt} = -\frac{1}{50}y(y - 100)$$

where $y = y(t)$ is the size of the population at time t and k is a positive constant.
Estimate the size of the population after a long period of time if the initial size is 90 wolves.
Justify your answer.

ANSWER: The size of the population after a long period of time is

Exercise 3 **[1+2+5+2+(1 bonus) points]**

Consider the matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$.

(a) Compute the trace and the determinant of $\mathbf{A}$.

ANSWERS: The trace of $\mathbf{A}$ is The determinant of $\mathbf{A}$ is

(b) Compute the characteristic polynomial of $\mathbf{A}$ and determine the eigenvalues of $\mathbf{A}$.

ANSWERS: Characteristic polynomial: Eigenvalues:

(c) Find the general solution of the system of differential equations $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

ANSWER:

(d) Find the solution of the initial value problem $\mathbf{x}' = \mathbf{A}\mathbf{x}$ with the initial condition $\mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

ANSWER:

Please turn $\longrightarrow$

(e) **Bonus question: 1 additional point**
The system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ is equivalent to a second order differential equation $ay'' + by' + cy = 0$.
Determine a, b and c.

ANSWER:

Exercise 4 [3 points]
Determine the largest interval I where the solution of the following initial value problem exists and is unique:

$$(e^t - 1)\frac{dy}{dt} + y = \frac{t}{t-2} \qquad \text{with the initial condition} \qquad y(1) = 2$$

Justify your work. *(Do not attempt to solve the differential equation)*

ANSWER: $I =$