Practical information on Midterm 2

- Midterm 2 is on Monday, April 6, 2020, 10:30-12:30 (Atlanta time).
- Midterm 2 will be released on Canvas $\longrightarrow$ Quizzes.
- Please email your scanned PDF solution by the deadline to: angela.pasquale@univ-lorraine.fr or angela.pasquale@georgiatech-metz.fr .
- Midterm 2 coverage: lectures and recitations on Chapter 3 (sections 3.1 to 3.6), Chapter 6 (sections 6.1 to 6.4), Chapter 4 (sections 4.1 to 4.5, 4.7).
- For the Midterm, *you will need to know:* the material and the examples from the lectures, and the exercises covered in the recitations. The solution sheets for the recitations made in March are available on this webpage. Please be sure to review them. Below you will find *some* additional review problems you might want to try *if* you have extra time. They only touch some part of the material covered by the Midterm.
- In the Midterm you have to solve the problems by yourself, you are not allowed to discuss problems and solutions with other people in any form. Please abide to the Honor Code.
- I will be online during the whole exam time. You can send me messages by email. I will do my best to answer as soon as I can. Sometimes I will be answering to other people. So please be patient. Also, please understand that there are questions to which I cannot answer: for instance, if your solution is correct or not.

Two additional problems

Exercise 1 For each of the following systems of differential equations:

1. Find the general solution in terms of real-valued functions.
2. Solve the initial value problem with initial condition $x_1(0) = -1$, $x_2(0) = 1$.
3. Determine the equilibrium points, identify their type, determine their stability.
4. Determine the behaviour of the solutions for $t \to +\infty$ and for $t \to -\infty$.
5. Can you sketch some trajectories in the phase portrait?

(a) $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$; (b) $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} \mathbf{x}$;

(c) $\mathbf{x}' = \begin{pmatrix} -1/2 & 3 \\ 0 & 2 \end{pmatrix} \mathbf{x}$; (d) $\mathbf{x}' = \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix} \mathbf{x}$;

Exercise 2 (a) Determine the longest interval in which the following IVP is certain to have a unique twice differentiable solution:

$$2t^2y'' + 3ty' - y = 0, \qquad y(\overset{1}{\cancel{0}}) = 0, y'(\overset{1}{\cancel{0}}) = 1\,.$$

(Do not attempt to find the solution.) Justify your answer.

(b) Verify that $y_1(t) = t^{1/2}$ and $y_2(t) = t^{-1}$ are two solutions of the differential equation $2t^2y''+3ty'-y = 0$ for $t > 0$.

(c) Show that y_1 and y_2 are linearly independent solutions of $2t^2y'' + 3ty' - y = 0$ for $t > 0$.

(d) Find the general solution of $2t^2y'' + 3ty' - y = 0$ for $t > 0$.

(e) Find the general solution of the non-homogenous differential equation $2t^2y'' + 3ty' - y = t$ for $t > 0$.

(f) Convert $2t^2y'' + 3ty' - y = t$, $t > 0$, into an equivalent system of first-order differential equations. Write your answer in matrix form.

(g) Determine the general solution of the system found in (f).