Georgia Tech – Lorraine
Spring 2020
Differential Equations
Math 2552
2/12/2020

Last Name:
First Name:

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Midterm n^0 2 (2 hours)

- Please email your solution to *angela.pasquale@univ-lorraine.fr* or *angela.pasquale@georgiatech-metz.fr* today, by 12:30 pm (Atlanta time). Write *"Midterm 2"* in the subject.
- You can write your solution on the same pdf I sent you, for instance if you have a tablet or you can print it. If you need extra space, add as many pages as needed.
  Otherwise, please write your solutions on blank paper. You do not need to copy the questions, just clearly *mark the number of the exercises and their questions* and *separate the different exercises with a horizontal line.*
- Please call your file "yourname-Midterm2".
- Show your work and justify your answers. Please organize your work clearly, neatly, and legibly. Identify your answers.
- You have to solve the problems by yourself, you are not allowed to discuss problems and solutions with other people in any form. Please abide to the Honor Code.
- I will be online during the whole exam time. You can send me messages by email. I will do my best to answer as soon as I can. Sometimes I will be answering to other people. So please be patient. Also, please understand that there are questions to which I cannot answer: for instance, if your solution is correct or not.
- Maximum: 25 points

Exercise 1 [3+1+1+5+1 points]

Consider the linear differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0$ where $t > 0$.

(a) Verify that $y_1(t) = t$ and $y_2(t) = te^t$ are solutions and that they are linearly independent when $t > 0$.

Please turn: Questions (b) to (e) on the following page ⟶

Exercise 1 (continued)

(b) Write the general solution of $t^2y'' - t(t+2)y' + (t+2)y = 0$ for $t > 0$.

We now consider the non-homogenous differential equation

$$t^2y'' - t(t+2)y' + (t+2)y = t^4e^t\,. \tag{1}$$

(c) Write it in standard form.

(d) Find a particular solution.

(e) Determine the general solution of (1).

Exercise 2 [3+1+1+1 points]

Consider the system of linear DE's $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 5 & -3 \\ 3 & -5 \end{pmatrix}$.

(a) Find the general solution.

(b) Determine the equilibrium point, identify its type and determine its stability.

Please turn: Questions (c) to (d) on the following page $\longrightarrow$

Exercise 2 (continued)

(c) Pick one of the two eigenvalues of $\mathbf{A}$ you determined in (a) and call it λ. Find an eigenvector $\mathbf{v}$ of $\mathbf{A}$ for the eigenvalue λ so that the solution $\mathbf{x}(t) = e^{\lambda t}\mathbf{v}$ has first component equal to 1 at $t = 0$.

(d) Let $\mathbf{x}(t) = e^{\lambda t}\mathbf{v}$ the solution you determined in (c). Sketch its trajectory in the phase plane.
(Do not forget to indicate by an arrowhead the direction of motion along the trajectory).

Please turn $\longrightarrow$

Exercise 3 **[1+3+3+1 points]**

A mass of 2 kg is hung from a spring of spring constant $k = 1.85$ N/m. Suppose that it is also attached to a viscous damper that exerts a force of 0.03 N when the velocity of the mass is 0.05 m/s. The mass is pulled down 0.1 m below its equilibrium position and then released. Suppose that there is no external force.

(a) Determine the damping coefficient γ.

(b) Write down the appropriate initial value problem that governs the motion of the mass.

Please turn: Questions (c) to (d) on the following page $\longrightarrow$

Exercise 3 (continued)

(c) Solve the initial value problem and find the position of the mass at any time t.

(d) Determine the quasi-period of the motion.