Review sheet for Midterm 1

Chapter 1: Introduction

Section 1.1: use differential equations for modeling. Initial value problems (IVP).

Section 1.2 and Section 2.5: Autonomous differential equations: find equilibrium solutions (also called critical points or stationary points), draw phase lines, sketch integral curves. Determine if a critical point is asymptotically stable, semistable or unstable. (drawing of direction fields is not requested).

Section 1.3: classification of differential equations: order, linear/non linear, homogeneity of linear equations.

Chapter 2: First order differential equations

Section 2.1: solve separable equations.

Section 2.2: standard form of a first order linear DE. Solve first order linear differential equations by using integrating factors.

Section 2.3: modeling: write down a differential equation to model a problem and then solve the differential equation (or the IVP).

Section 2.4: existence and uniqueness of solutions: first order linear DE (Theorem 2.4.1) and first order non-linear DE (Theorem 2.4.2).

Section 2.5: see Section 1.2.

Section 2.6: recognize a first order exact DE and solve it.

Section 2.7: recognize a first order DE with homogenous coefficients and solve it. Recognize a Bernoulli DE and solve it.

Chapter 3: Systems of two first order equations

Section 3.1: Systems of two linear equations. Homogenous systems. Matrix notation. Matrix of coefficients of the system. Trace and determinant of a 2×2 matrix. Invertible matrices. A matrix is invertible if and only if its determinant is non-zero. Inverse of a matrix. Solutions of linear systems. Characteristic polynomial and characteristic equation. Eigenvalues and eigenvectors.

Section 3.2: Systems of two first-order linear DE's. Solutions. Initial value problems (IVP). Theorem on the existence and uniqueness of the solutions of an IVP for a system of two linear DE's (Theorem 3.2.1). Matrix notation, vector solution. Special cases: homogenous systems; systems with constant coefficients
Transform a second order linear DE into a system of first order linear DE's.

Section 3.3: Homogeneous systems $\mathbf{x}' = \mathbf{A}\mathbf{x}$ with constant coefficients.
The superposition principle (Theorem 3.3.1), Wronskian and linear independence, notion of fundamental system of solutions, general solution (Theorem 3.3.4).
Constructing solutions using eigenvalues and eigenvectors of $\mathbf{A}$.
The general solution when $\mathbf{A}$ admits two linearly independent eigenvectors.