Review sheet for Midterm 2

Chapter 3: Systems of two first order equations

Section 3.1: Systems of two linear equations. Homogenous systems. Matrix notation. Matrix of coefficients of the system. Trace and determinant of a 2×2 matrix. Invertible matrices. A matrix is invertible if and only if its determinant is non-zero. Inverse of a matrix. Solutions of linear systems. Characteristic polynomial and characteristic equation. Eigenvalues and eigenvectors.

Section 3.2: Systems of two first-order linear DE's. Solutions. Initial value problems (IVP). Theorem on the existence and uniqueness of the solutions of an IVP for a system of two linear DE's (Theorem 3.2.1). Matrix notation, vector solution. Special cases: homogenous systems; systems with constant coefficients.
Transform a second order linear DE into a system of first order linear DE's.

Section 3.3: Homogeneous systems $\mathbf{x}' = \mathbf{A}\mathbf{x}$ with constant coefficients.
The superposition principle (Theorem 3.3.1), Wronskian and linear independence, notion of fundamental system of solutions, general solution (Theorem 3.3.4).
Constructing solutions using eigenvalues and eigenvectors of $\mathbf{A}$.
The general solution when $\mathbf{A}$ admits two linearly independent eigenvectors.

For a homogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x}$: Superposition principle (Theorem 3.3.1), Wronskian and linear independence, fundamental sets of solutions, general solution (Theorem 3.3.4).

Sections 3.3, 3.4 and 3.5 (analytic methods): Solve the homogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x}$:

• Find a fundamental system of solutions and write the general solution (depending of the nature of the eigenvalues of $\mathbf{A}$).

• When $\mathbf{A}$ has complex (conjugate) eigenvalues, write the solution in terms of real solutions (Section 3.4).

• Behavior of the solutions when $\mathbf{A}$ has two distinct real eigenvalues (Table 3.3.1) or complex eigenvalues (Table 3.4.1) or repeteated eigenvalues (Table 3.5.1).

Sections 3.2, 3.3, 3.4 and 3.5 (geometric methods): Component plots of solutions. Autonomous systems: notions of phase plane, trajectories, direction fields, equilibrium point (or equilibrium point or critical point), phase portrait.

For a homogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x}$:

• Determine if $(0, 0)$ is a nodal sink, nodal source, saddle, spiral sink, spiral source, or a center. Stability.

• Behavior of the solutions when $\mathbf{A}$ has two distinct real eigenvalues (Table 3.3.1) or complex eigenvalues (Table 3.4.1) or repeteated eigenvalues (Table 3.5.1).

• Sketch some trajectories in the phase plane.

Sections 3.6: nonlinear systems of two differential equations. Vector notation. Existence and uniqueness of the solutions (Theorem 3.6.1). Autonomous systems (see also Chapter 7).

Chapter 6: Systems of first-order linear equations

Section 6.1: Systems of n linear first-order DE's; matrix notation. The system of first order linear DE's associated with a linear n-th order differential equation; correspondence of initial value problems in this context.

Section 6.2: Existence and unicity of solutions for systems of first-order linear equations.
For a homogeneous system $\mathbf{x}' = \mathbf{A}\mathbf{x}$: Superposition principle (Theorem 6.2.2), Wronskian and linear independence, fundamental sets of solutions, general solution (Theorem 6.2.6).

Section 6.3: Defective and nondefective matrices. The general solution of a linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A}$ is nondefective with real eigenvalues (Theorem 6.3.1)

Section 6.4: Real-valued general solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A}$ is nondefective and not all eigenvalues of $\mathbf{A}$ are real.

Chapter 4: Second order linear equations

Section 4.1: Second order linear equations: standard form, homogeneous/nonhomogenous, constant coefficients/variable coefficients, initial value problems (IVP). See below for spring-mass system models.

Section 4.2: The system of first order linear differential equations associated with a second order linear differential equation, correspondence of initial conditions, matrix notation.
Existence and uniqueness of the solutions of an IVP for a 2nd order linear DE (Theorem 4.2.1).
Second order linear homogenous DE's: principle of superposition for a 2nd order DE (Theorem 4.2.2, Corollary 4.2.3) and for a homogenous system of 1st order linear DE's (Theorem 4.2.4, Corollary 4.2.5). Wronskian of two solutions, fundamental solutions and general solution: for homogenous systems of two 1st order linear DEs (Theorem 4.2.6) and for 2nd order linear DE (Theorem 4.2.7).

Section 4.3: Second order linear homogenous DE's with constant coefficients: characteristic equation, fundamental system of solutions constructed from the roots of the characteristic equation (Theorem 4.3.1, for the DE and for its associated system), general solution (Theorem 4.3.2).

Section 4.5: Solutions of a second order linear nonhomogenous DE: the general solution as a sum of the general solution of the corresponding homogenous DE (complementary solution) and one particular solution (Theorems 4.5.1 and 4.5.2). The method of undetermined coefficients for finding a particular solution when the corresponding homogenous differential equation has constant coefficients.

Section 4.7: The method of variation of parameters.

Sections 4.1 and 4.4: Spring-mass systems
- *Section 4.1:* the model: mass, spring constant, damping factor.
- *Section 4.4:* unforced or free systems (harmonic oscillators).

Undamped free system: phase-amplitude form of the general solution (period, natural frequency, phase, amplitude).
Damped free system: underdamped, critically damped or overdamped harmonic motion; critical damping; quasi-frequency and quasi-period of an underdamped harmonic motion.
Phase portraits for harmonic oscillators.