Georgia Tech – Lorraine
Fall 2019
Differential Equations
Math 2552
8/29/2019

Last Name:
First Name:

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Quiz n^0 1 (20 minutes)

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise 1 (4+3+3 points) .
The liquid base of an ice cream at initial temperature of 30° C is placed in a freezer with constant temperature −20° C. Assume that Newton's law of cooling applies with transmission factor $k = 0.5$ hours^{-1}.

(a) Write an initial value problem (IVP) that models the temperature of the liquid as a function of time.

Solution: Set:
t = time (in hours)
$u(t)$ = temperature (in degrees C) of the liquid at the time t
$u(0)$ = initial temperature of the liquid=30° C
T_0 = temperature of the freezer=−20° C
By Newton's law of cooling with transmission factor $k = 0.5$ hours^{-1} we obtain the IVP

$$\frac{du}{dt} = -k(u - T_0) = -0.5(u + 20)\,, \qquad u(0) = 30$$

(b) Solve the IVP and determine a formula for the temperature of the liquid as a function of time.

Solution: The constant function $u = -20°$ C is a solution of the DE, but not the solution we are looking for, as $u(0) = 30°$ C. Suppose then $u \neq -20$. Divide both sides of the DE by $u + 20$ and get:

$$\frac{1}{u+20}\,\frac{du}{dt} = -0.5$$

Integrate with respect to t:

$$\int \frac{1}{u+20}\,\frac{du}{dt}dt = -0.5\int dt \quad \text{i.e.} \quad \int \frac{1}{u+20}\,du = -0.5\int dt$$

and obtain $\ln|u+20| = -0.5t + C_0$ (where C_0 is a constant). Exponentiating gives the general solution $u(t) = -20 + Ce^{-0.5t}$ (where C is a constant).

We determine C from the initial condition $u(0) = 30$ by inserting $t = 0$ in the general solution: $30 = u(0) = -20 + C$ gives $C = 50$.

Conclusion: $u(t) = -20 + 50e^{-0.5t}$ (in degrees C).

(c) Suppose the ice cream is ready when it reaches the temperature of 0° C. How long will it take to be ready? *(Leave your answer in term of* ln*)*

Solution: We look for the time t_0 for which $u(t_0) = 0°$ C:

$$0 = u(t_0) = -20 + 50e^{-0.5t_0} \quad \text{yields} \quad 5e^{-0.5t_0} = 2, \quad \text{i.e.} \quad e^{0.5t_0} = \frac{5}{2}$$

. Thus: $0.5t_0 = \ln\left(\frac{5}{2}\right)$, i.e. $t_0 = 2\ln\left(\frac{5}{2}\right)$ hours.

Please turn ⟶

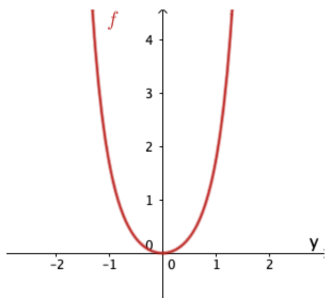
Exercise 2 (3+2+5 points) . The differential equation

$$\frac{dy}{dt} = e^{y^2} - 1$$

is of the form $\frac{dy}{dt} = f(y)$ with $f(y) = e^{y^2} - 1$

(a) Sketch the graph of $f(y)$ versus y

Solution:



(b) Determine the equilibrium point(s).

Solution: $f(y) = 0$ if and only if $y = 0$. So $y = 0$ is the unique equilibrium point.

(c) Draw the phase line and classify the equilibrium point(s) as asymptotically stable, unstable, or semistable. Sketch graphs of solutions in the ty-plane on either sides of each equilibrium point.

Solution:

$y' = f(y) > 0$ for all $y \neq 0$.

The equilibrium point $y = 0$ is semistable.