$\frac{dQ}{dt}$ = {rate of salt in} − {rate of salt out}

{rate of salt flow} = {rate of water flow} · {salt concentration in water flow}

$$\text{lb/min} \quad = \quad \text{gal/min} \quad \cdot \quad \text{lb/gal}$$

IN : $10 \text{ gal/min} \cdot 0 \text{ lb/gal} = 0$ [pure water ⇒ no salt concentration]

OUT : $10 \text{ gal/min} \cdot \underbrace{\frac{Q(t)}{100}}_{\text{salt concentration (lb/gal)}} = \frac{1}{10} Q(t)$

salt concentration (lb/gal) = {= amount of salt per gallon, the volume of liquid in the tank being constant}

Conclusion : $\begin{cases} \frac{dQ}{dt} = -\frac{1}{10} Q(t) \\ Q(0) = 20 \end{cases}$

Equilibrium solutions are solutions to $\frac{dQ}{dt} = 0$

i.e. $-\frac{1}{10} Q = 0$, i.e. $Q = 0$ (unique)

The function $f(Q) = -\frac{1}{10} Q$ is $\begin{cases} >0 & \text{for } Q<0 \\ <0 & \text{for } Q>0 \end{cases}$

Q=0  (phase line). Thus $Q=0$ is asymptotically stable

There will be no salt in the tank : $\lim_{t \to +\infty} Q(t) = 0$.

No matter what is the initial concentration of salt, for $t \to +\infty$ the solution $Q(t)$ tends asymptotically to the unique stable equilibrium solution $Q = 0$.

Please turn ⟶