

Last Name:
First Name:

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Quiz n^o 2 (20 minutes)

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise 1 (5+3+2 points) .

Pure water is poured into a tank at the rate of 10 gal/min. Initially, the tank contains 20 pounds of salt in 100 gallons of water. The contents of the tank are kept thoroughly mixed and flow out from the tank at the same rate of 10 gal/min.

Let $Q(t)$ denote the amount (in pounds) of salt in the tank at time t (in min).

- (a) Write an initial value problem (IVP) that models the rate of change of $Q(t)$ as a function of time.

- (b) Determine the unique equilibrium solution of the above system. Draw the phase line and check that it is asymptotically stable.

- (c) What happens to $Q(t)$ after a very long time t ? Justify your answer using (b)

Exercise 2 (1+3+4+2 points) . Consider the linear differential equation

$$\frac{dy}{dt} = 3t^2y - 6t^2$$

with initial condition $y(1) = 1$.

(a) Write the differential equation in standard form.

(b) Find an integrating factor.

(c) Find the general solution of the differential equation.

(d) Find the solution of the given initial value problem.