Georgia Tech – Lorraine
Fall 2019
Differential Equations
Math 2552
10/10/2019

Last Name:
First Name:

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Quiz n^0 3 (20 minutes)

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise 1 (2+3+2+3 points) .

Consider the system of differential equations $\begin{cases} \dfrac{dx}{dt} = -x + 1 \\ \dfrac{dy}{dt} = -2y \end{cases}$

(a) Suppose $x \neq 1$. Determine a first order differential equation for y as a function of x.

(b) Solve the differential equation in (a) and determine a function $H(x, y)$ such that every solution satisfies an equation of the form $H(x, y) = C$, where C is a constant.
(Write $H(x, y)$ so that is does not contain any logarithmic terms.)

(c) Describe the level curves of the function $H(x, y)$ and sketch some of them.

(d) For $t > 0$, sketch the trajectory corresponding to the initial condition $x(0) = 2$ and $y(0) = -2$ and indicate the direction of motion for increasing t.
(Sketch the trajectory only and not the level curve to which it belongs.)

Please turn $\longrightarrow$

Exercise 2 (2+4+2+2 points) .
A 1-kilogram mass stretches a spring 20 cm. The mass is pulled down 5 cm below its equilibrium position and given an initial upward velocity of 10 cm/s. Assume that there is no damping and recall that $g = 9.8\text{m/sec}^2$

(a) Determine the spring constant of this spring.

(b) Write an initial value problem (IVP) that models the motion of the mass.

(Choose a downward-pointing coordinate axis with origin at the equilibrium position. Do not solve this IVP)

(c) Introduce state variables and convert the IVP of (b) into an IVP for a system of two first-order linear differential equations. Use matrix notation.

(Do not solve this IVP)

(d) Will the system oscillate indefinitely? Explain.
(A mathematical argument is expected.)