Georgia Tech – Lorraine
Fall 2019
Differential Equations
Math 2552
10/24/2019

Last Name:
First Name:

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Quiz n^0 4 (20 minutes)

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise. [Each of the questions (a),(b),(c),(d) and (e) below is worth 4 points.]
An object of mass $m = 0.2$ kg is hung from a spring with spring constant $k = 40$ N/m. The object is subject to a damping with damping coefficient $\gamma = 4$ Ns/m.

(a) Suppose first that there is no external force acting on the spring-mass system.

Set up the differential equation of the motion.

(Choose the equilibrium point as the origin of a downward-pointing y-coordinate axis.)

(b) Determine the general solution of the differential equation you found in part (a).

(c) Suppose that at time $t = 0$ the mass is pulled down 0.5 m below its equilibrium position and then released (i.e. the initial velocity is 0).

Determine the motion $y(t)$ of the mass as a function of the time t.

Please turn $\longrightarrow$

(d) The following two questions consider the motion from a qualititative point of view:

(d1) Can the function $y(t)$ be written in the form $y(t) = h(t)\cos(\nu t - \delta)$ for a suitable function $h(t)$ and suitable constants ν and δ? If so, determine $h(t)$, ν and δ.

(d2) What is the behavior of $y(t)$ as t increases?

(e) Suppose now that the mass-spring system is subject to a periodic force $F(t) = 100\sin(20t)$ N. Explain (*without computing it!*) how one can find a steady-state solution. Your explanation must include an initial guess for the particular solution.