Georgia Tech – Lorraine
Spring 20
Differential Equations
Math 2552
16/1/2020

Last Name:
First Name:

EX	
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2	
TOT	

Quiz n^0 1 (20 minutes)

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise 1 (4+3+3 points) .
The temperature of a cake when it is removed from the oven is 150° C. The cake is left in a room at the constant temperature of 20° C. Five minutes later its temperature is 80° C.
Assume that Newton's law of cooling applies with transmission factor k (in min^{-1}).

(a) Write an initial value problem (IVP) that models the temperature of the cake as a function of time. *(You need not determine the value of k.)*

Set: t = time (in min); $t=0$ when the cake removed from the oven
$u(t)$ = temperature of the cake at the time t (in °C); $u(0) = 150\,°C$
T = room temperature = 20° C

By Newton's law of cooling with transmission factor k, the temperature of the cake is modeled by the IVP : $\frac{du}{dt} = -k(u-20)$ with $u(0)=150$

(b) Solve the IVP and determine a formula for the temperature of the cake as a function of the time t and of the transmission factor k.

(Do not determine the value of k.)

$\frac{du}{du} = -k(u-20)$ is a 1st order separable DE. $u=20$ is its unique constant solution.

If $u \neq 20$, can divide both sides by $u-20$ and get: $\frac{1}{u-20}\frac{du}{dt} = -k$

Integrate both sides wrt t and recall that $\frac{du}{dt}dt = du$

$\int \frac{1}{u-20}\frac{du}{dt}dt = -k\int dt$, i.e. $\int \frac{1}{u-20}du = -k\int dt$, which yields $\ln|u-20| = -kt + C_0$, C_0 constant

Exponentiate both sides: $|u-20| = e^{C_0}e^{-kt}$, i.e. $u-20 = \pm e^{C_0 - kt}$

Hence $u(t) = 20 + ce^{-kt}$, c = arbitrary real constant (equal to $\pm e^{C_0}$ if $u \neq 20$, and to $c=0$ for the constant solution $u=20$)

$150 = u(0) = 20 + C$. So $C = 130$. Thus $u(t) = 20 + 130e^{-kt}$

(c) Determine the value of the k (in min^{-1}).

(Leave your answer in term of ln*)*

The measurement $u(5) = 80°C$ yields
$80 = 20 + 130e^{-5k}$, i.e. $\frac{6}{13} = e^{-5k} \Rightarrow e^{5k} = \frac{13}{6}$, i.e. $k = \ln\left(\frac{13}{6}\right)\cdot\frac{1}{5}$

Please turn ⟶

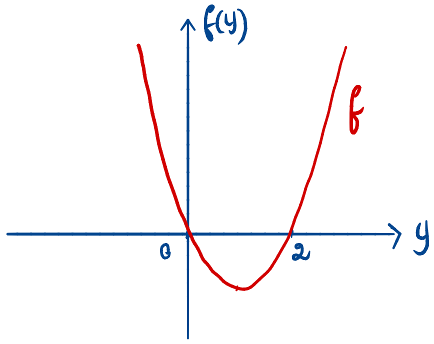
Exercise 2 (3+2+5 points) . The differential equation

$$\frac{dy}{dt} = y(y-2)$$

is of the form $\frac{dy}{dt} = f(y)$ with $f(y) = y(y-2)$.

(a) Sketch the graph of $f(y)$ versus y.

$f(y) = y(y-2)$



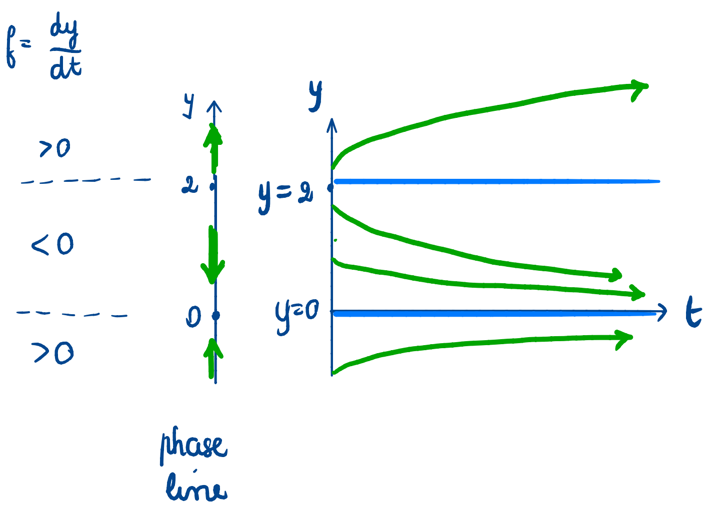
(b) Determine the equilibrium point(s).

$f(y) = 0 \iff y = 0$ or $y = 2$.

The equilibrium points (= constant solutions) are therefore

$y = 0$ and $y = 2$.

(c) Draw the phase line and classify the equilibrium point(s) as asymptotically stable, unstable, or semistable. For $t \geq 0$, sketch graphs of solutions in the ty-plane on either sides of the equilibrium point(s).



$y=2$ is an unstable equilibrium point

$y=0$ is an asymptotically stable eq. point