Georgia Tech – Lorraine Spring 20 Differential Equations Math 2552 16/1/2020

Last Name: First Name: EX 1 2 TOT

Quiz n^0 1 (20 minutes)

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise 1 (4+3+3 points).

The temperature of a cake when it is removed from the oven is 150° C. The cake is left in a room at the constant temperature of 20° C. Five minutes later its temperature is 80° C. Assume that Newton's law of cooling applies with transmission factor k (in min⁻¹).

(a) Write an initial value problem (IVP) that models the temperature of the cake as a function of time. (You need not determine the value of k.)

(b) Solve the IVP and determine a formula for the temperature of the cake as a function of the time t and of the transmission factor k.

(Do not determine the value of k.)

(c) Determine the value of the k (in min⁻¹). (Leave your answer in term of \ln) Exercise 2 (3+2+5 points). The differential equation

$$\frac{dy}{dt} = y(y-2)$$

is of the form $\frac{dy}{dt} = f(y)$ with f(y) = y(y-2).

(a) Sketch the graph of f(y) versus y.

(b) Determine the equilibrium point(s).

(c) Draw the phase line and classify the equilibrium point(s) as asymptotically stable, unstable, or semistable. For $t \ge 0$, sketch graphs of solutions in the *ty*-plane on either sides of the equilibrium point(s).