Georgia Tech - Lorraine Spring 20
Differential Equations
Math 2552
$16 / 1 / 2020$

| Last Name: |
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| 1 |  |
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## Quiz $n^{0} 1(20$ minutes $)$

Show your work and justify your answers. Calculators, notes, cell phones, books are not allowed. Please do not use red or pink ink. Maximum: 20 points

Exercise 1 ( $4+3+3$ points) .
The temperature of a cake when it is removed from the oven is $150^{\circ} \mathrm{C}$. The cake is left in a room at the constant temperature of $20^{\circ} \mathrm{C}$. Five minutes later its temperature is $80^{\circ} \mathrm{C}$.
Assume that Newton's law of cooling applies with transmission factor $k\left(\right.$ in $\left.\mathrm{min}^{-1}\right)$.
(a) Write an initial value problem (IVP) that models the temperature of the cake as a function of time. (You need not determine the value of $k$.)
(b) Solve the IVP and determine a formula for the temperature of the cake as a function of the time $t$ and of the transmission factor $k$.
(Do not determine the value of $k$.)
(c) Determine the value of the $k$ (in $\min ^{-1}$ ).
(Leave your answer in term of $\ln$ )

Exercise $2(3+2+5$ points) . The differential equation

$$
\frac{d y}{d t}=y(y-2)
$$

is of the form $\frac{d y}{d t}=f(y)$ with $f(y)=y(y-2)$.
(a) Sketch the graph of $f(y)$ versus $y$.
(b) Determine the equilibrium point(s).
(c) Draw the phase line and classify the equilibrium point(s) as asymptotically stable, unstable, or semistable. For $t \geq 0$, sketch graphs of solutions in the $t y$-plane on either sides of the equilibrium point(s).

