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### Quiz n<sup>o</sup> 3 (make-up)

- *Please email your solution to [angela.pasquale@univ-lorraine.fr](mailto:angela.pasquale@univ-lorraine.fr) or [angela.pasquale@georgiatech-metz.fr](mailto:angela.pasquale@georgiatech-metz.fr) by 12:15 pm (Atlanta time). Write “Quiz 3” in the subject.*
- *You can write your solution on the same pdf I sent you, for instance if you have a tablet or you can print it. If you need extra space, add as many pages as needed. Otherwise, please write your solutions on blank paper. You do not need to copy the questions, just clearly mark the number of the exercise and separate the different exercises with a horizontal line.*
- *Show your work and justify your answers. Please organize your work clearly, neatly, and legibly. Identify your answers.*
- *The solution of this quiz must be your own. Do not show, discuss or compare your solution with anybody else.*
- *During the time interval from the release time and submission deadline, I will be online. If you have questions about the quiz, you can send me email messages or Canvas messages.*
- *Maximum: 20 points.*

**Exercise 1 (6 points)** Transform the initial value problem  $y'' - y' = e^t$  with initial conditions  $y(0) = 1$  and  $y'(0) = 2$ , into an equivalent initial value problem for a first order system. Write your answer in matrix form.

**Exercise 2 (7 points)** .

Consider the homogeneous system of linear DE's:  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ , where  $\mathbf{x} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$  and  $\mathbf{A}$  is a  $3 \times 3$  matrix with real entries.

Suppose that the following are pairs of eigenvalues and corresponding eigenvectors of  $\mathbf{A}$ :

$$\bullet \lambda_1 = -1, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix},$$

$$\bullet \lambda_2 = 1, \quad \mathbf{v}_2 = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix},$$

$$\bullet \lambda_3 = 2, \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

Write a fundamental set of solutions of  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ .

Explain why the solutions are linearly independent.

**Exercise 3 (7 points)** . Consider the homogeneous system of linear DE's:  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ , where  $\mathbf{x} = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$  and  $\mathbf{A}$  is a  $2 \times 2$  matrix with real entries.

Suppose that  $\lambda_1 = 2 - i$  is a complex eigenvalue of  $\mathbf{A}$  with corresponding eigenvector  $\mathbf{v} = \begin{pmatrix} 2 \\ 3i \end{pmatrix}$ .

Determine the general solution of  $\mathbf{x}' = \mathbf{A}\mathbf{x}$  in terms of real-valued functions.