Georgia Tech – Lorraine
Spring 2020
Differential Equations
Math 2552
3/29/2020

Last Name:
First Name:

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Quiz n^0 4

- Due Saturday, April 4, at noon (Atlanta time).
- The solution of the quiz must be your own. You may not show, discuss or compare your solutions with anyone else.
- Please return your quiz by email to:

  angela.pasquale@univ-lorraine.fr or angela.pasquale@georgiatech-metz.fr

  Write "Quiz 4" in your email's subject.
  Receipt will be acknowledged by email.
- Please check that your scanned solution is readable.
- Coverage of this quiz: Chapter 3 and Chapter 4, sections 4.1 to 4.3.
- Please do not use red or pink ink. If you are use a pencil, be sure that it is dark enough.
- Maximum: 20 points

In this quiz you will be asked to use the MIT Mathlet, Linear Phase Portraits: Matrix Entry, available at: https://mathlets.org/mathlets/linear-phase-portraits-matrix-entry/

Some indications about the MIT Mathlet "Linear Phase Portraits: Matrix Entry":

When the `[Companion Matrix]` option is selected, the first row entries of the displayed matrix are fixed to be 0 and 1. By deselecting the `[Companion Matrix]` option, you can choose all four entries a, b, c, d of the matrix. The values of a, b, c, d can be fixed between -4 et 4 using the corresponding sliders.
The point in the upper-left window gives the determinant and the trace of the displayed matrix.
If you select the `[eigenvalues]` option, the eigenvalues of the matrix become visible: their values are displayed and their location is plotted in the complex plane.
The big window on the upper-right corner of the screen shows the phase plane of the system (the coordinates are denoted by x and y instead of x_1 and x_2 as in the lectures). It displays the trajectories of a few solutions.
Placing the cursor on a point of the phase plane displays its (x, y)-coordinates below the bottom left corner of the phase plane. Clicking produces the trajectory passing through that point. You can clear all the trajectories using `[Clear]`, and return to the original set of trajectories by re-setting one of the sliders for the matrix entries.

Exercise 1 (4+3+3=10 points) Consider the system of linear DE's $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix}$ and $\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.

(a) Determine its general solution.

(b) Enter the matrix $\mathbf{A}$ into the MIT Mathlets application. A few trajectories are displayed in the phase portrait. A trajectory crosses the x-axis at $x = 2$. What is the solution having this as a trajectory assuming that this crossing occurs at $t = 0$?

(c) Write the equation of the solution $\mathbf{x}(t)$ whose trajectory is the half-line in the 3^{rd} quadrant (i.e. where $x < 0$, $y < 0$) and so that $\mathbf{x}(0) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

Please turn $\longrightarrow$

Exercise 2 (4+3+3 points) . Consider second-order linear differential equation $x'' - 4x' + 3x = 0$ where $x = x(t)$ is the unknown function.

(a) Find its general solution.

(b) Consider the associated dynamical system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ (i.e. the associated system of linear DE). Enter the matrix $\mathbf{A}$ into the MIT Mathlets application. A trajectory crosses the x-axis at $x = 2$. What is the solution of $x'' - 4x' + 3x = 0$ which corresponds to this trajectory if we assume that this crossing occurs at $t = 0$?

(c) Sketch in the phase plane the trajectory corresponding to the solution $x(t) = e^t$ of $x'' - 4x' + 3x = 0$.