

Last Name:
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Quiz n^o 5

- *Please email your solution to angela.pasquale@univ-lorraine.fr or angela.pasquale@georgiatech-metz.fr by 12:15 pm (Atlanta time). Write “Quiz 5” in the subject. Double check that your file is readable and complete.*
- *You can write your solution on the same pdf I sent you, for instance if you have a tablet or you can print it. If you need extra space, add as many pages as needed. Otherwise, please write your solutions on blank paper. You do not need to copy the questions, just clearly mark the number of the exercise and separate the different exercises with a horizontal line.*
- *Show your work and justify your answers. Please organize your work clearly, neatly, and legibly. Identify your answers.*
- *The solution of this quiz must be your own. Do not show, discuss or compare your solution with anybody else.*
- *A table of Laplace transforms is at the end of this file. You can use the table on your textbook if you prefer.*
- *During the time interval from the release time and submission deadline, I will be online. If you have questions about the quiz, you can send me email messages.*
- *Maximum: 20 points.*

Please turn →

Exercise 1 (4+6 points) .

(a) Write the following function using the unit step function

$$f(t) = \begin{cases} t^2 & \text{if } 0 \leq t < \pi \\ \cos(2t) & \text{if } \pi \leq t < 2\pi \\ 0 & \text{if } t \geq 2\pi \end{cases}$$

(b) Find the Laplace transform of the function f in (a).

Exercise 2 (10 points) . Solve the following initial value problem using Laplace transforms.

$$y'' + 2y' + 5y = e^{-t} \sin t \quad \text{with initial conditions } y(0) = y'(0) = 0$$

Elementary Laplace transforms.

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1.	1	$\frac{1}{s}, \quad s > 0$	Sec. 5.1; Ex. 4
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 5.1; Ex. 5
3.	$t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 5.2; Cor. 5.2.5
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 5.1; Prob. 37
5.	$\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$	Sec. 5.1; Ex. 7
6.	$\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$	Sec. 5.1; Prob. 22
7.	$\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > a $	Sec. 5.1; Prob. 19
8.	$\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > a $	Sec. 5.1; Prob. 18
9.	$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$	Sec. 5.1; Prob. 23
10.	$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$	Sec. 5.1; Prob. 24
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 5.2; Prob. 8
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 5.5; Eq. (4)
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 5.5; Eq. (6)
14.	$e^{ct}f(t)$	$F(s-c)$	Sec. 5.2; Thm. 5.2.1
15.	$\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$	Sec. 5.6; Thm. 5.8.3
16.	$\delta(t-c)$	e^{-cs}	Sec. 5.7; Eq. (14)
17.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$	Sec. 5.2; Cor. 5.2.3
18.	$t^n f(t)$	$(-1)^n F^{(n)}(s)$	Sec. 5.2; Thm. 5.2.4