Math 2552, Fall 2019, recitation n7

Saturday, September 28, 2019 3:44 PM

6.4 Nondefective Matrices with Complex Eigenvalues

PROBLEMS

In each of Problems 1 through 8, express the general solution of the given system of equations in terms of real-valued functions:

1. $$\begin{aligned} x_1' &= -2x_1 + 2x_2 + x_3 \\ x_2' &= -2x_1 + 2x_2 + 2x_3 \\ x_3' &= 2x_1 - 3x_2 - 3x_3 \end{aligned}$$

5. $$\mathbf{x}' = \begin{pmatrix} -7 & 6 & -6 \\ -9 & 5 & -9 \\ 0 & -1 & -1 \end{pmatrix} \mathbf{x}$$

3.6 A Brief Introduction to Nonlinear Systems

PROBLEMS

For each of the systems in Problems 1 through 6:

(a) Find an equation of the form $H(x, y) = c$ satisfied by the solutions of the given system.

(b) Without using a computer, sketch some level curves of the function $H(x, y)$.

(c) For $t > 0$, sketch the trajectory corresponding to the given initial condition and indicate the direction of motion for increasing t.

1. $dx/dt = -x, \quad dy/dt = -2y; \qquad x(0) = 4, \; y(0) = 2$

6. $dx/dt = 2y, \quad dy/dt = -8x; \qquad x(0) = 1, \; y(0) = 2$

4.1 Definitions and Examples

For each spring-mass system or electric circuit in Problems 10 through 17, write down the appropriate initial value problem based on the physical description.

10. A mass weighing 2 lb stretches a spring 6 in. The mass is pulled down an additional 3 in. and then released. Assume there is no damping.

11. A mass of 100 g stretches a spring 20 cm. The mass is set in motion from its equilibrium position with a downward velocity of 5 cm/s. Assume there is no damping.

15. A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant of 2 lb·s/ft. The mass is set in motion from its equilibrium position with a downward velocity of 3 in./s.

4.2 Theory of Second Order Linear Homogeneous Equations

PROBLEMS

In each of Problems 1 through 8, determine the longest interval in which the given initial value problem is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

7. $(1-x^2)y'' - 2xy' + (\alpha(\alpha+1) + \mu^2/(1-x^2))y = 0,$
$y(0) = y_0, \quad y'(0) = y_1$

15. Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are two solutions of the differential equation $t^2y'' - 2y = 0$ for $t > 0$. Then show that $c_1t^2 + c_2t^{-1}$ is also a solution of this equation for any c_1 and c_2.