

# MATH 2552 recitation 7 solutions

Tuesday, March 10, 2020 6:01 PM

## Chapter 3

### Systems of Two First Order Equations

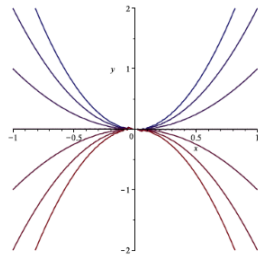
#### 3.6 A Brief Introduction to Nonlinear Systems

- 1.(a)  $\frac{dx}{dt} = -x$ ,  $\frac{dy}{dt} = -2y$ , thus  $\frac{dy}{dx} = \frac{2y}{x}$ , and then

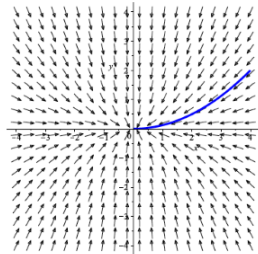
$$\int \frac{1}{y} dy = 2 \int \frac{1}{x} dx,$$

thus  $\ln |y| = 2 \ln |x| + C$ . Applying the exponential function to this equation, we conclude that  $y = Cx^2$  for any solution of the system. That is,  $H(x, y) = y/x^2$ .

(b)



(c)



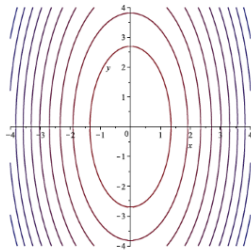
As  $t$  increases, the  $x$  and  $y$  values are decreasing.

- 6.(a)  $\frac{dx}{dt} = 2y$ ,  $\frac{dy}{dt} = -8x$ , thus  $\frac{dy}{dx} = \frac{-4x}{y}$ , and then

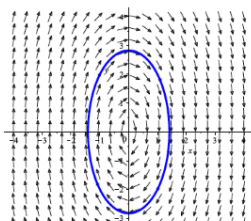
$$\int y dy = - \int 4x dx,$$

thus  $y^2/2 = -2x^2 + C$ . Therefore,  $y^2 + 4x^2 = C$  for any solution of the system. That is,  $H(x, y) = y^2 + 4x^2$ .

(b)



(c)





As  $t$  increases, the trajectory travels in the clockwise direction.

## Chapter 4

### Second Order Linear Equations

#### 4.1 Definitions and Examples

10. The spring constant is  $k = 2/(1/2) = 4$  lb/ft. The mass  $m = 2/32 = 1/16$  lb-s<sup>2</sup>/ft. The equation of motion is

$$\frac{1}{16}y'' + 4y = 0$$

or  $y'' + 64y = 0$ , with initial conditions  $y(0) = 1/4$  ft,  $y'(0) = 0$  ft/sec.